## The Goods Market and the IS Relation

We characterized equilibrium in the goods market as the condition that production, Y, be equal to the demand for goods, Z. We called this condition the IS relation. We defined demand as the sum of consumption, investment, and government spending. We assumed that consumption was a function of disposable income (income minus taxes), and took investment spending, government spending, and taxes.

The equilibrium condition in the goods market was given by: $Y = C(Y - T) + I + G$

The main simplification of this first model was that the interest rate did not affect the demand for goods. Our first task in this chapter is to abandon this simplification and introduce the interest rate in our model of equilibrium in the goods market.

## Investment, Sales, and the Interest Rate

In chapter3, we assumed investment was constant, even when output change. Investment is in fact far from constant, and it depends primarily on two factors:

- **The level of sales:** A firm facing an increase in sales needs to increase production. To do so, it may need to buy additional machines, or to build an additional plant. A firm facing low sales will feel no such need and will spend little, if anything on investment.
- **The interest rate:** Consider a firm deciding whether to buy a new machine. Suppose that to buy the new machine, the firm must borrow. The higher the interest rate, the less attractive it is to borrow and to buy the machine. At a high enough interest rate, the additional profit from using the new machine will not cover interest payment, and the new machine will not be worth buying.

To capture these two effects, we write the investment relation as follows:

$$I = \underset{(+,-)}{I\,(Y, i)}$$

The equation states that investment (I) depends on production (Y), and the interest rate (i).

We continue to assume that inventory investment is equal to zero, so sales and production are always equal. As a result, (Y) denotes sales and it also denotes production.

The positive sign under Y indicates that an increase the production (sales) leads to an increase in investment. The negative sign under the interest rate ($i$) indicates that an increase in the interest rate leads to a decrease in investment.

## The Determination of Output

Taking into account the investment relation, the condition for equilibrium in the goods market becomes:

$$Y = C(Y - T) + I\,(Y, i) + G$$

## IS relation:

Production (the left side of the equation) must be equal to the demand for goods (the right side). The equation: $Y = C(Y - T) + I\,(Y, i) + G$, *is* our expanded **IS relation**.

*What happens to output when the interest rate changes?*

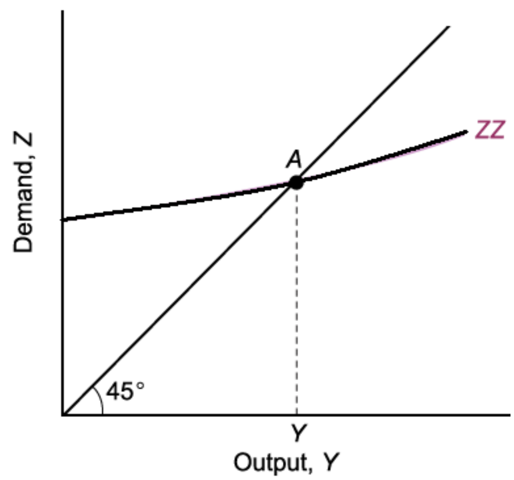
For a given value of the interest rate $i$, demand is an increasing function of output, for two reasons:

- An increase in output leads to an increase in income and thus to an increase in disposable income. The increase in disposable income leads to an increase in consumption.
- An increase in output also leads to an increase in investment.

## Equilibrium in the Goods Market

In short, an increase in output leads, through its effects on both consumption and investment, to an increase in the demand for goods. This relation between demand and output, for a given interest rate, is represented by the upward-sloping curve $ZZ$.

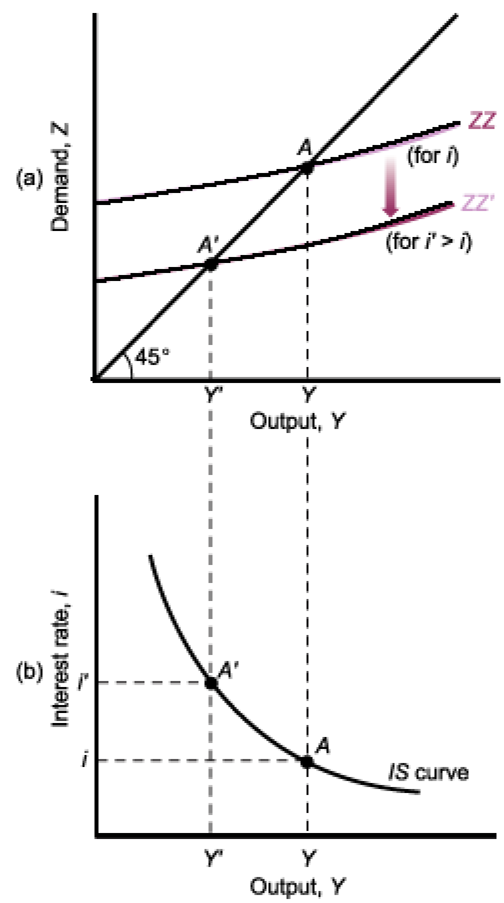
Note two characteristics of $ZZ$ *in* the Figure




- Since we have not assumed that the consumption and investment relations in the IS relation are linear, $ZZ$ is in general a curve rather than a line. Thus, we have drawn it as a curve in the Figure.
- We have drawn $ZZ$ so that it is flatter than the 45-degree line. Put another way, we have assumed that an increase in output leads to a less than one-for-one increase in demand. (Slope of $ZZ$ curve equal to c1, (c1 <1)).

Equilibrium in the goods market is reached at the point where the demand for goods equals output; that is, at point *A*, the intersection of $ZZ$ and the 45-degree line. The equilibrium level of output is given by *Y*.

## Deriving the IS Curve




Suppose that the demand curve is given by ZZ, and the initial equilibrium is at point A. suppose now that the interest rate increases from its initial value (i) to a new higher value (i'). At any level of output, the higher interest rate implies a decrease in investment, so a decrease in demand. The demand curve ZZ shifts down to ZZ'. The new equilibrium is at the intersection of the lower demand curve ZZ' and the 45-degree line, so at point A'. The equilibrium level of output is now equal to Y'.

In words: The increase in the interest rate decrease investment. The decrease in investment leads to a decrease in output, which further decrease consumption and investment.

- At panel (a), the interest rate (i) implies a level of output equal to Y. The higher interest rate to (i'), implies a lower level of output at (Y').
- At panel (b) plots equilibrium output Y on the horizontal axis against the interest rate on the vertical axis. Point A in panel (b) corresponds to point A in panel (a), and point A' in panel (b) corresponds to A' in panel (a).

This relation between the interest rate and output is represented by downward sloping curve in panel (b). This curve is called the IS curve.

## Shifts of the IS Curve:

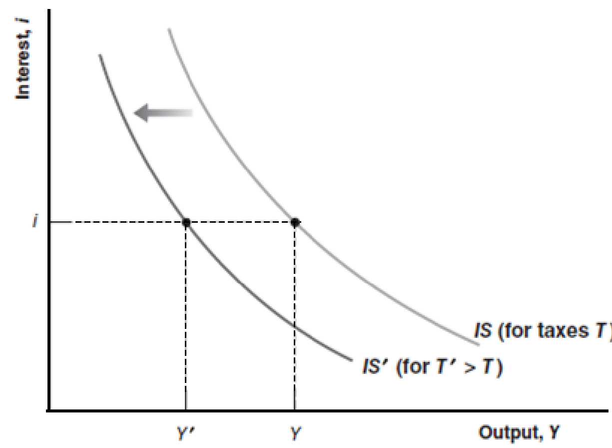
IS curve is drawn for given values of taxes ($T$), and government spending ($G$), and consumer confidence ($c0$). Changes in either T or G will shift the IS curve.

Consider an increase in taxes from T to T'. At a given interest rate, say (i), disposable income decrease, leading to a decrease in consumption, leading in turn to decrease demand for goods and a decrease in equilibrium output from Y to Y'. This lead to shift IS curve to the left.

- A decrease in taxes (T), an increase in government spending (G), an increase in consumer confidence $c0$, leads the IS curve to shift to the right.

- An increase in taxes (T), a decrease in government spending (G), a decrease in consumer confidence $c0$, leads the IS curve to shift to the left.

More generally, any factor that, for a given interest rate, decreases the equilibrium level of output causes the IS curve to shift to the left. We have looked at an increase in taxes. But the same would hold for a decrease in government spending, or a decrease in consumer confidence (which decreases consumption given disposable income).




Symmetrically, any factor that, for a given interest rate, increases the equilibrium level of output—a decrease in taxes, an increase in government spending, an increase in consumer confidence—causes the IS curve to shift to the right.

✓ Changes in factors that decrease the demand for goods given the interest rate shift the IS curve to the left. Changes in factors that increase the demand for goods given the interest rate shift the IS curve to the right.

## Financial Markets and the LM Relation

The equilibrium in financial market implies that money demand equal money supply: $M = \$Y\ L(i)$

The variable M on the left side is the nominal money stock (money supply). The central bank controlling M directly. The right side gives the demand for money, which is a function of nominal income ($Y), and of the nominal interest rate (i).

The right side gives the demand for money, which is a function of nominal income, $Y, and of the nominal interest rate, i. An increase in nominal income increases the demand for money; an increase in the interest rate decreases the demand for money. Equilibrium requires that money supply (the left side of the equation) be equal to money demand (the right side of the equation).

## Real Money, Real Income, and the Interest Rate

The equation $M = \$Y\ L(i)$ gives a relation between money, nominal income, and the interest rate. Rewrite this equation as a relation between real money (money in terms of goods), real income (income in terms of goods), and the interest rate.

Dividing both sides of the equation by the price level (P), gives:

$$\frac{M}{P} = Y\ L(i)$$

(From chapter 2: nominal GDP = real GDP multiplied by the GDP deflator: $Y = P Y).

we can restate our equilibrium condition as the condition that the real money supply–that is, the money stock in terms of goods, not dollars–be equal to the real money demand, which depends on real income, Y, and the interest rate, i.

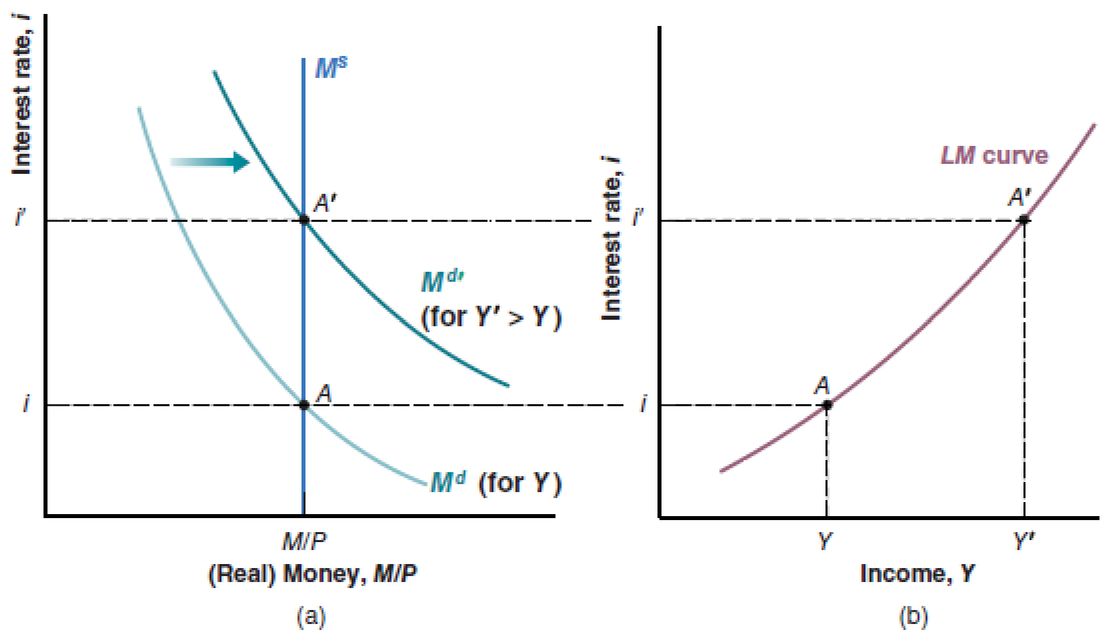
LM relation: $\frac{M}{P} = Y\,L(i)$ ("real money supply" equal "real money demand)

## Deriving the LM Curve

Consider an increase in income from Y to Y', which leads people to increase their demand for money at any given interest rate. Money demand shifts to the right, to $M^{d'}$. The new equilibrium is at point A', with a higher interest rate (i').

Panel (b) plots the equilibrium interest rate (i) on the vertical axis against income on the horizontal axis. Point A in panel (b) corresponds to point A in panel (a). Point A' in panel (b) corresponds to point A' in panel (a).

- Equilibrium in financial markets implies that an increase in income leads to an increase in the interest rate. The LM curve is upward sloping.




**Example:**
Imagine an economy that can be described with the following equations:

$C = 500 + 0.8(Y - T)$

$I = 200 + 0.10Y - 5i$

$G = 100 \quad T = 100$

Money Demand = $M^d = 0.5Y - 100i$

Money Supply = $M^S = 1000$

Derive the IS-LM relations

IS relation: equilibrium in goods market

$Z = C + I + G = 500 + 0.8\,(Y - 100) + 200 + 0.1Y - 5i + 100$

$Z = 720 + 0.8Y + 0.1Y - 5i$

$Z = 720 + 0.9Y - 1000i$

$At\ equilibrium: Y = Z$

$Y = 720 + 0.9Y - 5i \Rightarrow 0.1Y = 720 - 5i$

$$Y = \frac{720 - 5i}{0.1}$$

| Y = 7,200 - 50 i ------------------ IS relation |
|---|

LM relation: equilibrium in financial market

At equilibrium: real money demand = real money supply

$0.5Y - 100i = 1{,}000 \rightarrow 0.5Y = 1{,}000 + 100i$

$$\Rightarrow Y = \frac{1{,}000 + 100i}{0.5}$$

| Y = 2,000 + 200i ---------------- LM relation |
|---|

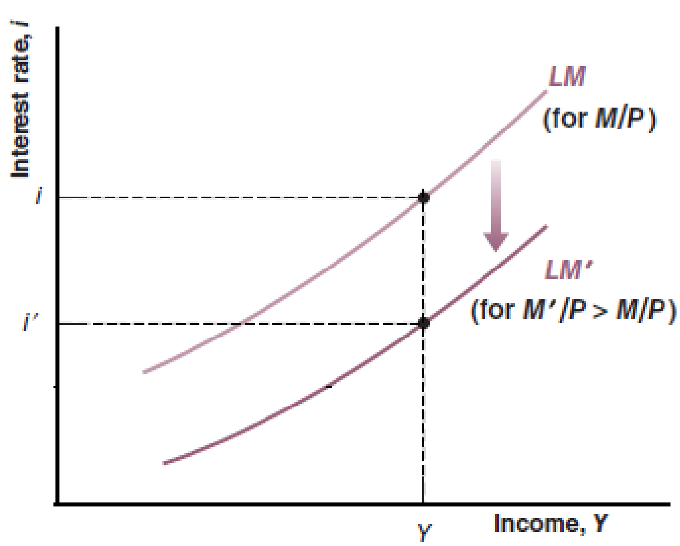
## Shifts of the LM Curve

Changes in M/P, whether they come from changes in nominal money stock (M), or from changes in the price level (P), will shift the LM curve

Consider an increase in the nominal money supply (M), from M to M', so that, at the same price level, the real money supply increase from M/P to M'/P. Then, at any level of income, the interest rate is lower. The LM curve shifts down, from LM to LM'.

- ✓ Increase in money supply shift the LM curve down (increase); decrease in the money supply shift the LM curve up (decrease).

- ✓ An increase in the price level shift LM curve up; decrease in the price level shift LM curve down.

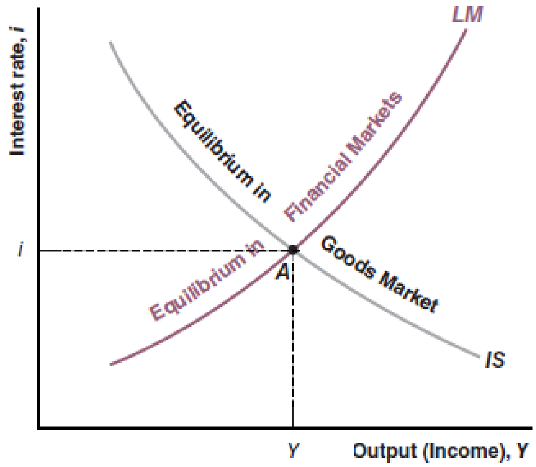



## Putting the IS and the LM Relations Together

IS relation: $Y = C(Y - T) + I(Y, i) + G$

LM relation: $\frac{M}{P} = Y\,L(i)$

Any point on the downward sloping IS curve corresponds to equilibrium in the goods market. Any point on the upward sloping LM curve corresponds to equilibrium in financial markets. Only at point A are both equilibrium conditions satisfied.




## Fiscal Policy, Output, and the Interest Rate

Suppose the government decides to reduce the budget deficit, and does so by increasing taxes while keeping government spending unchanged. Such a change in fiscal policy is often called a fiscal contraction.

Fiscal contraction: increase in taxes (T ↑) or a decrease in government spending (G↓)

An increase in the deficit, either due to an increase in government spending or to a decrease in taxes, is called a fiscal expansion.

Fiscal expansion: decrease in taxes (T↓) or an increase in government spending (G↑)

*What are the effects of fiscal contraction on output, and interest rate?*

An increase in taxes shifts IS curve to the left. LM curve does not shift.

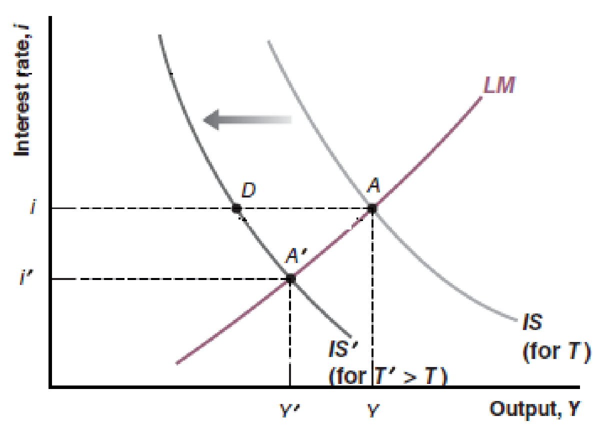
Output (Y): decrease from Y to Y'

Interest rate (i) : decrease from i to i'

What happen to the components of demand (C and I)?

Consumption (C) : decrease ( Y ↓ → $C$ ↓)

Investment (I): $\left\{\begin{matrix} Y \downarrow \rightarrow I \downarrow \\ i \downarrow \rightarrow I \uparrow \end{matrix}\right\}$ Net effect : Investment : uncertain)



The case where investment falls as the deficit rises is sometimes called the **crowding out** of investment by the deficit. If investment instead rises when the deficit rises, there is **crowding in** of investment by the deficit.)

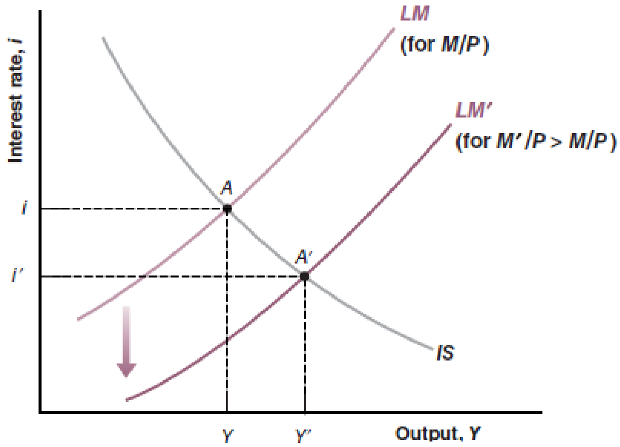
## Monetary Policy, Output, and the Interest Rate

An increase in the money supply is called a *monetary expansion*. A decrease in the money supply is called a *monetary contraction or monetary tightening*.

*What are the effects of monetary expansion on output, and interest rate?*

Suppose that the central bank increases nominal money (M), through an open market operation. Given our assumption that the price level is fixed.

An increase in money supply will shift LM curve down from LM to LM'. IS curve does not shift.




Output (Y): increase from Y to Y'

Interest rate (i) : decrease from i to i'

What happen to the components of demand (C and I)?

Consumption (C) : Increase ( $Y \uparrow \rightarrow C \uparrow$)

Investment (I): $\begin{Bmatrix} Y \uparrow \rightarrow I \uparrow \\ i \downarrow \rightarrow I \uparrow \end{Bmatrix}$ Net effect : Investment ↑)

## Fiscal policy multiplier, monetary policy multiplier, and tax multiplier

Consider the following IS-LM model:

$C = c_0 + c_1(Y - T)$

$I = b_0 + b_1 Y - b_2 i$

$M^d = d_1 Y - d_2 i$

*G, T, and $M^s$ are given*

Equilibrium in the goods market: $Y = Z = C + I + G$

$Y = c_0 + c_1(Y - T) + b_0 + b_1 Y - b_2 i + G$

$Y = c_0 + c_1 Y - c_1 T + b_0 + b_1 Y - b_2 i + G$

$Y - c1Y - b_1 Y = c0 + b_0 - c_1 T - b_2 i + G$ ------------- *(1)*

Equilibrium in financial market: $M^d = M^s$

$M^s = d_1 Y - d_2 i$

$d_2 i = d_1 Y - Ms$

$i = \frac{d1}{d2} Y - \frac{\text{Ms}}{d2}$ ------------ *(2)*

Solve equations *(1)* and *(2):*

$Y - c_1 Y - b_1 Y = c_0 + b_0 - c_1 T - b_2 ((d_1/d_2) Y - M^s/d_2) + G$

$Y - c_1 Y - b_1 Y = c_0 + b_0 - c_1 T - \frac{b2*d1}{d2} Y + \frac{b2}{d2} M^s + G$

$Y - c_1 Y - b_1 Y + \frac{b2*d1}{d2} Y = c_0 + b_0 - c_1 T + \frac{b2}{d2} M^s + G$

$Y (1 - c_1 - b_1 + \frac{b2*d1}{d2}) = c_0 + b_0 - c_1 T + \frac{b2}{d2} M^s + G$

$Y = \frac{1}{1 - c1 - b1 + \frac{b2*d1}{d2}} [c_0 + b_0 - c_1 T + \frac{b2}{d2} M^s + G]$

Fiscal policy multiplier $(m) = \frac{1}{1 - c1 - b1 + \frac{b2*d1}{d2}}$

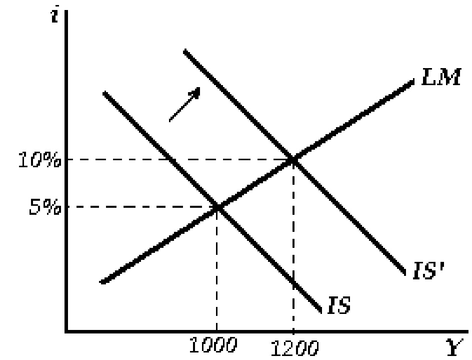
Monetary policy multiplier $(m_m) = \frac{b_2}{d_2} \left\{ \frac{1}{1 - c1 - b1 + \frac{b2*d1}{d2}} \right\}$

Tax multiplier $(m_t) = \frac{-c_1}{1 - c1 - b1 + \frac{b2*d1}{d2}}$

## Example

Consider the following IS-LM model:

$C = 200 + 0.25\,(Y - T)$
$I = 150 + 0.25Y - 1000i$
$G = 250, \quad T = 200$
$(M/P)^d = 2Y - 8000i$
$M/P = 1600$




a. Derive the IS and LM relations.

LS relation: equilibrium in goods market.

Z = C + I + G = 200 + 0.25 (Y - 200) + 150 + 0.25Y - 1000i + 250
Z = 550 + 0.25Y + 0.25Y - 1000i
Z = 550 + 0. 5Y - 1000i

At equilibrium: Y = Z

Y = 550 + 0. 5Y - 1000i ⇨ 0.5Y = 550 - 1000i

| Y = 1,100 - 2000i ----------------- IS relation |
|---|

LM relation: equilibrium in financial market

At equilibrium: real money demand = real money supply ($(M/P)^d$ = M/P)

2Y - 8000i = 1,600 ⇨ 2Y = 1600 + 8000i

| Y = 800 + 4000i -------------- LM relation |
|---|

b. Solve for equilibrium output and interest rate.

At equilibrium: IS relation = LM relation

1,100 - 2000i = 800 + 4000i

⇨ 6000i = 300 ⇨ i = 300/6000 = 5%

From LM relation: Y = 800 + 4000 (0.05) ⇨ Y = 1,000

c. Solve for equilibrium values of C, I, and real money demand.

C = 200 + 0.25 (Y - T) = 200 + 0.25(1000 - 200) = 400
I = 150 + 0.25Y - 1000i = 150 + 0.25(1000) - 1000(0.05) = 350
$(M/P)^d$ = 2Y - 8000i = 2(1000) - 8000(0.05) = 1,600

d. Suppose that government spending increase to G= 400. Summarize the effects of an expansionary fiscal policy on Y, i and I.

Δ Y = m * ΔG

$$\text{Multiplier} = \frac{1}{1 - c1 - b1 + \frac{b2 * d1}{d2}} = \frac{1}{1 - 0.25 - 0.25 + \frac{1000*2}{8000}}$$

$$\text{Multiplier} = \frac{1}{0.75} = 1.334$$

Δ Y = 1.334 * (400 - 250) = 200

New output = 1000 + 200 = 1200

(Increase in G will increase output)

By using LM relation: Y = 800 +4000i ⇨ 1200 = 800 + 4000i

⇨ 4000i = 400 ⇨ i = 400/4000 = 10%.

$I = 150 + 0.25Y - 1000i = 150 + 0.25(1200) - 1000(0.10) = 350$

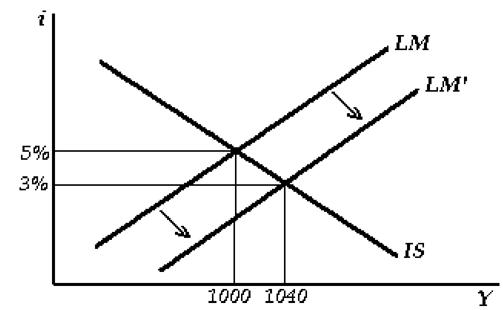
Output increase from Y = 1,000 to Y = 1,200
Interest rate increase from i = 5% to I = 10%
Investment unchanged (I = 350)

e. Set G equal to its initial value of 250. Now suppose that the money supply increase to M/P = 1840. Summarize the effects of an expansionary fiscal policy on Y, and i.

Δ Y = m * ΔG (m: monetary policy multiplier)

$$m = \frac{b_2}{d_2}\left\{\frac{1}{1 - c1 - b1 + \frac{b2*d1}{d2}}\right\} = \frac{1000}{8000}(1.334) = 0.16675$$

$\Delta Y = 0.6675 * (1840 - 1600) = 40$




New output = $1000 + 40 = 1{,}040$

By using IS relation: $Y = 1{,}100 - 2000i$ ⇨ $1{,}040 = 1{,}100 - 2000i$

$$i = \frac{(1{,}100 - 1{,}040)}{2000} = 0.03 = 3\%$$

$I = 150 + 0.25Y - 1000i = 150 + 0.25(1{,}040) - 1000(0.03) = 380$

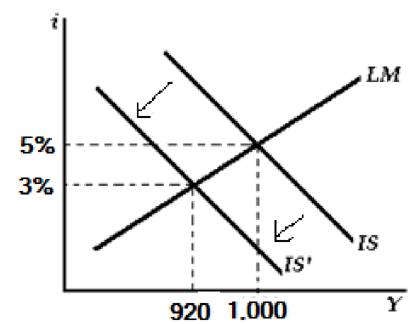
Output increase from Y = 1,000 to Y = 1,040
Interest rate decrease from i = 5% to I = 3%
Investment increase form I = 350 to I = 380

f. Set G equal to its initial value of 250 and M/P = 1,600. Now suppose that taxes increase to T = 440. Summarize the effects of an expansionary fiscal policy on Y, and i.

Δ Y = $m_t$ * ΔT

$$(m_t) = \frac{-c_1}{1 - c1 - b1 + \frac{b2*d1}{d2}} = -(0.25)(1.334) = -0.3335$$

Δ Y = $-0.3335$ * (440 - 200) = $-80$




New output = $1000 - 80 = 920$

By using LM relation: $Y = 800 + 4000i$ → $920 = 800 + 4000i$

$$i = \frac{(920 - 800)}{4000} = 0.03 = 3\%$$